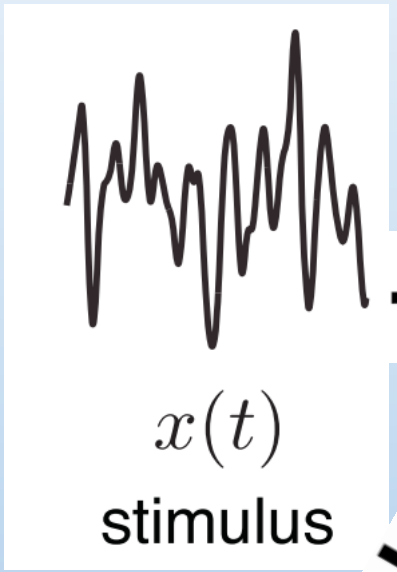
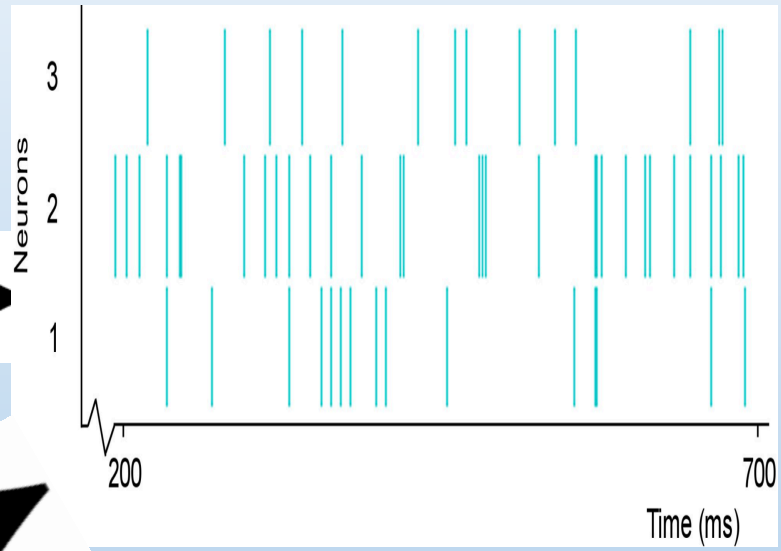
Generalized Linear Models

… and their generalizations!

Goal: Connect stimulus with spike train



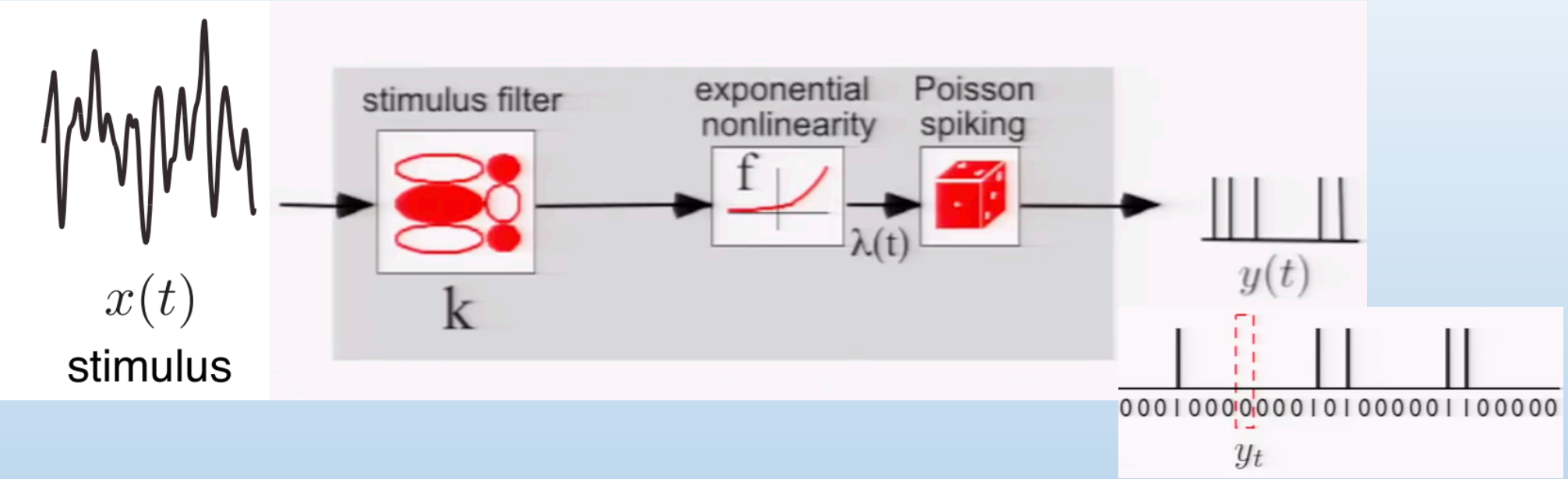


Light Intensity

Sound Frequency

Generalized Linear Models

GLM with Single Neuron



3 stages:

1) Linear Transformation (k initially unknown) $k \cdot x(t)$
2) Non-Linear $f(k \cdot x(t))$
3) Poisson spiking $\lambda(t) = f(k \cdot x(t))$

Finding stimulus filter

$$P(Y|\theta) = \prod_t \frac{(\lambda_t \Delta)^{y_t}}{y_t!} \exp(-\lambda_t \Delta)$$

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Finding stimulus filter

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$$\log P(Y|\theta) = \sum_t y_t \log \lambda_t + \sum_t y_t \log \Delta - \sum_t \log y_t! - \Delta \sum_t \lambda_t$$

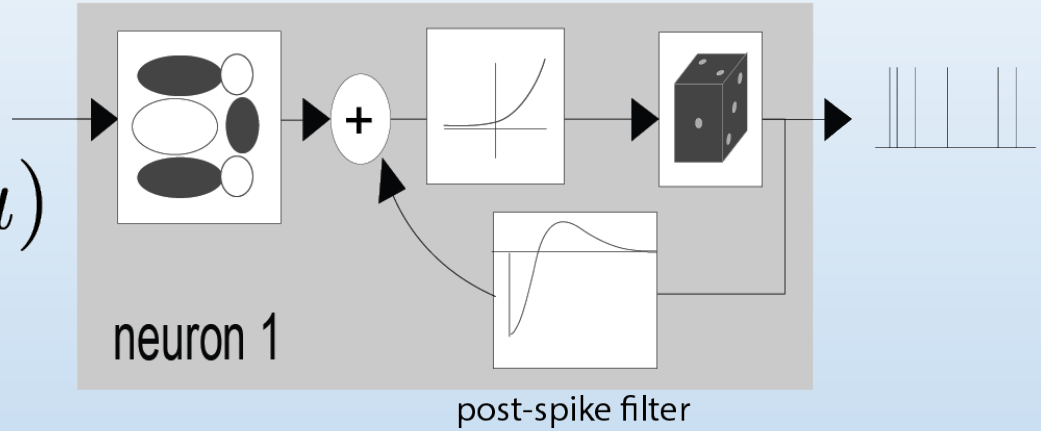
$$\mathcal{L}(\theta) = \sum_t y_t \log \lambda_t - \Delta \sum_t \lambda_t + c.$$

$$\lambda_t = f(\vec{k} \cdot \vec{x}_t + \mu)$$

Maximize Likelihood function by Gradient Ascent

Linear Post-Spike Filter

$$\lambda_t = f(\vec{k} \cdot \vec{x}_t + \vec{h} \cdot \vec{y}_t + \mu)$$

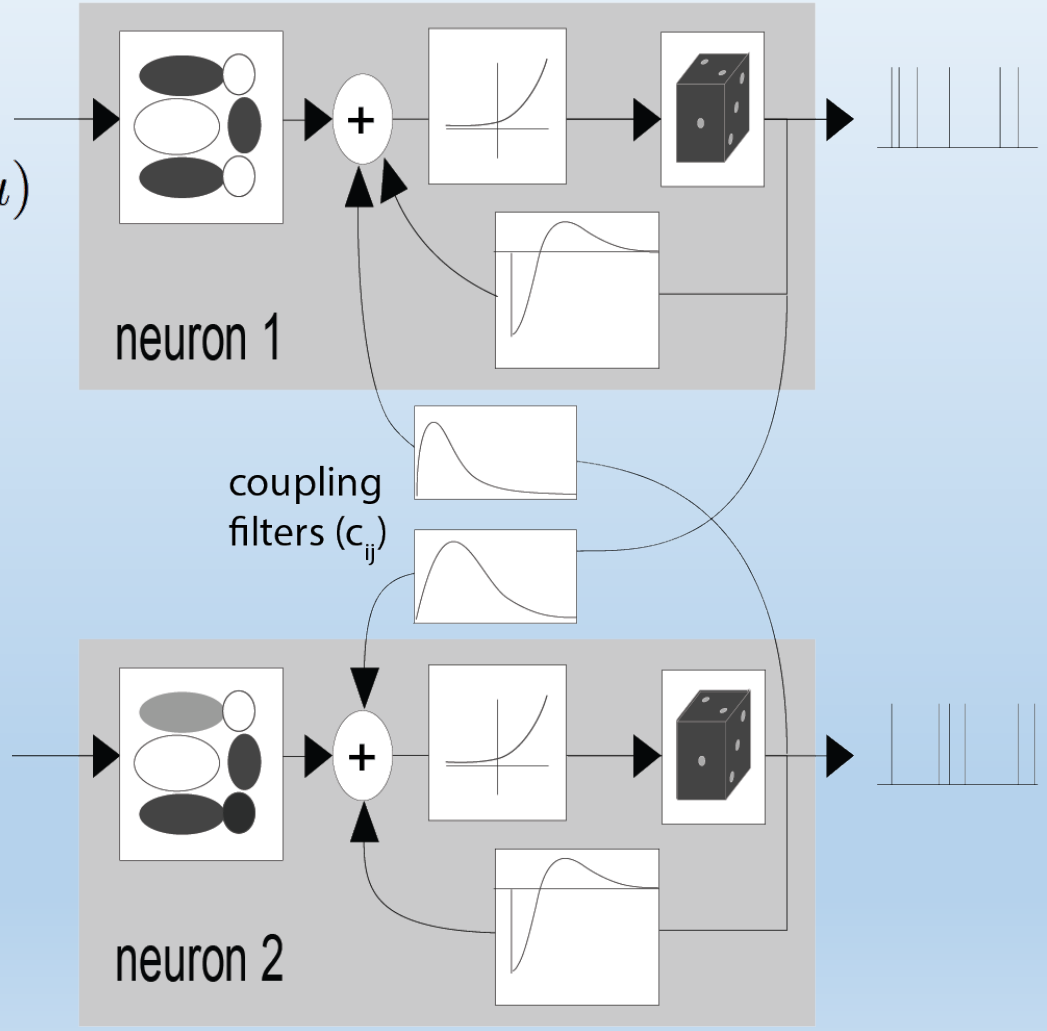


- Simulates refractory period

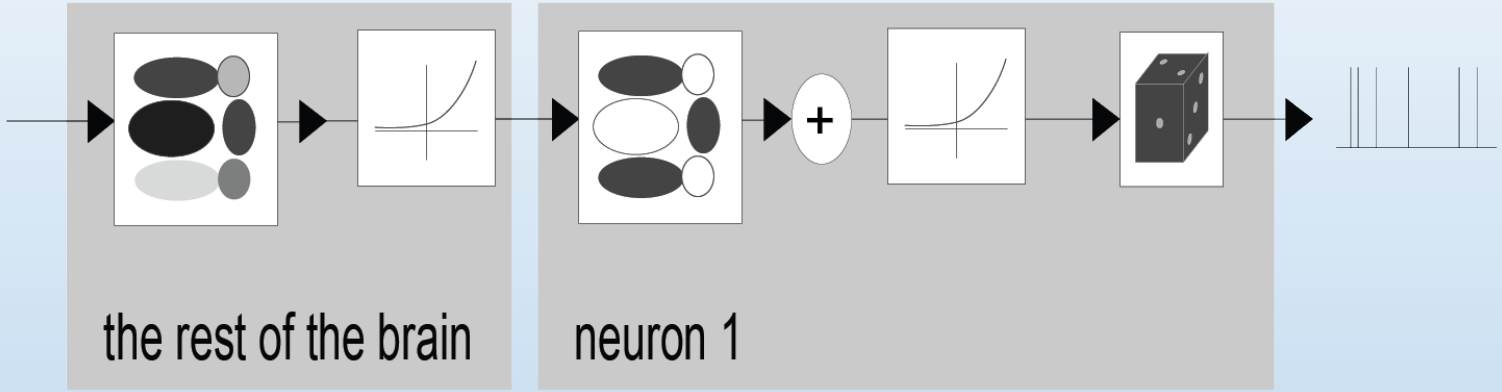
Linear Spike Coupling Filters

$$\lambda_t = f(\vec{k} \cdot \vec{x}_t + \vec{h} \cdot \vec{y}_t + \sum_i \vec{c}_i \cdot \vec{y}_{i,t} + \mu)$$

- Nearby neurons should be coupled



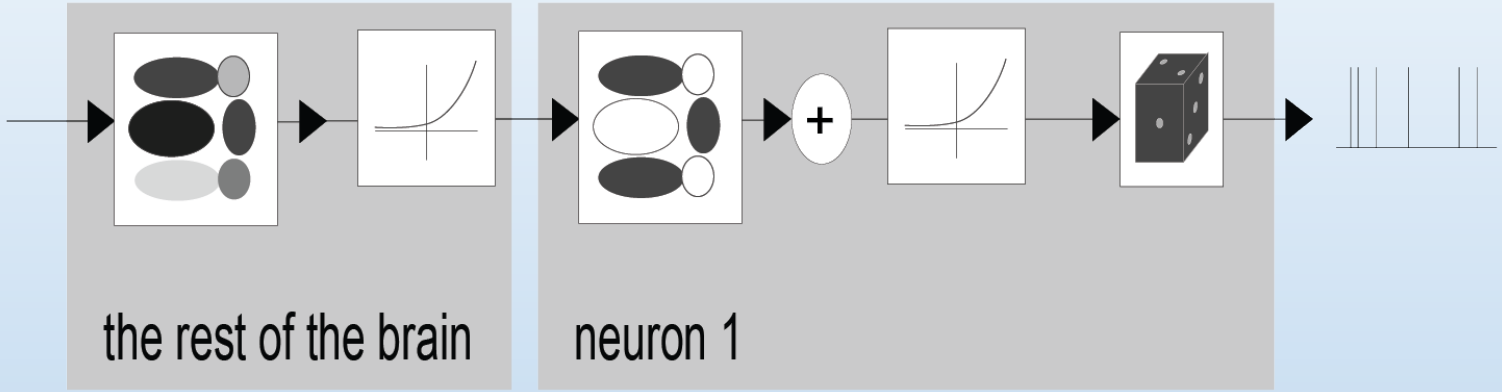
LNLNP structure



$$\lambda_t = f\left(\sum_i k_i f\left(\vec{K}_i \cdot \vec{x}_t\right) + \vec{h} \cdot \vec{y}_t + +\mu\right)$$

- Preprocessing between the sense neurons and the recorded neurons is almost certainly nonlinear
- Additional nonlinearity can serve as a proxy for the response of the rest of the brain?

LNLNP structure



$$\lambda_t = f\left(\sum_i k_i f\left(\vec{K}_i \cdot \vec{x}_t\right) + \vec{h} \cdot \vec{y}_t + +\mu\right)$$

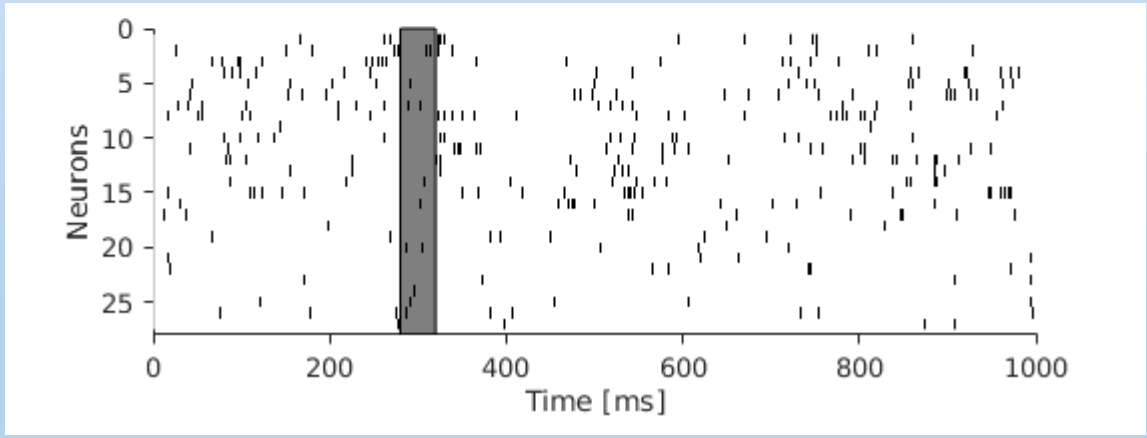
- Preprocessing between the sense neurons and the recorded neurons is almost certainly nonlinear
- Additional nonlinearity can serve as a proxy for the response of the rest of the brain?
- Need dimensional reductions to keep things managable

Meet the data

- 28 Retinal Ganglion Cells (RGCs) from isolated retina from macaque monkey
  - 28 arrays with spike times (15,000 to 52.000 spikes each)
- Stimulus: Black and white random checkerboard stimulus shown at 120Hz for 20 minutes
  - Matrix: 144,000 x 100 with 0/1 values

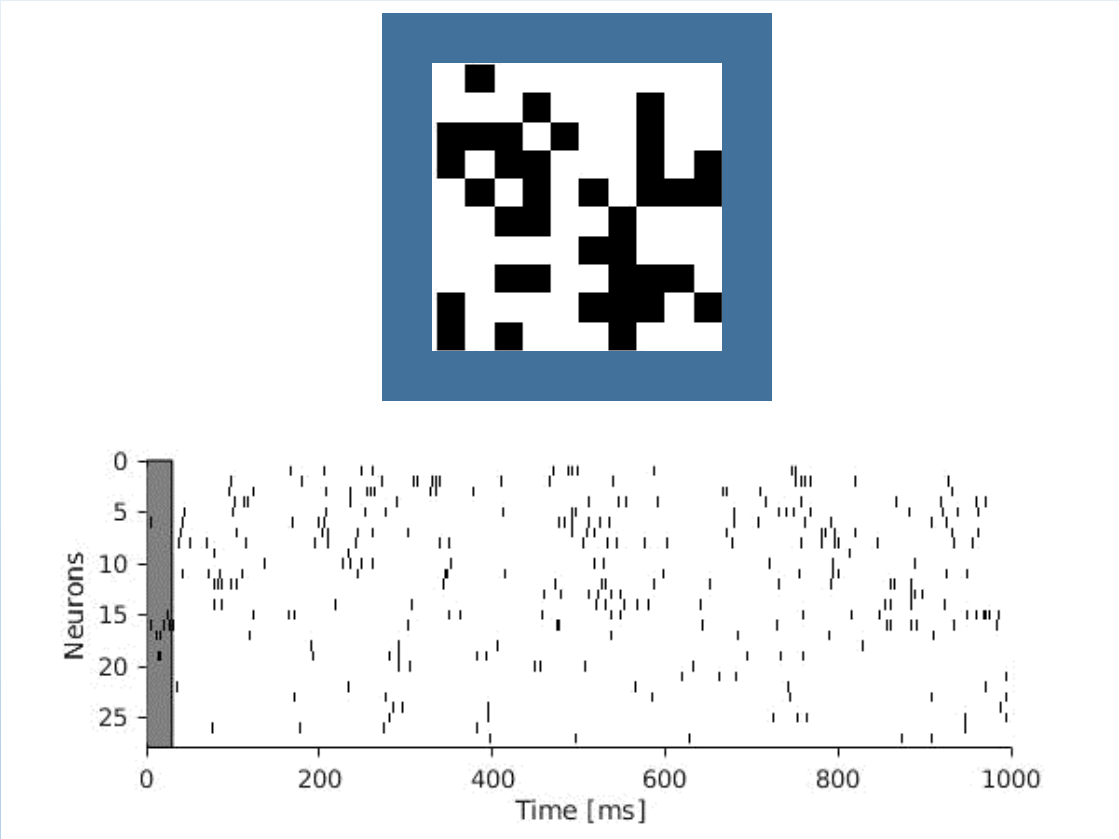
Example stimulus

Spike raster plot:



Data from Pillow et al., 2008, Natur
Available for all C3N summer studer

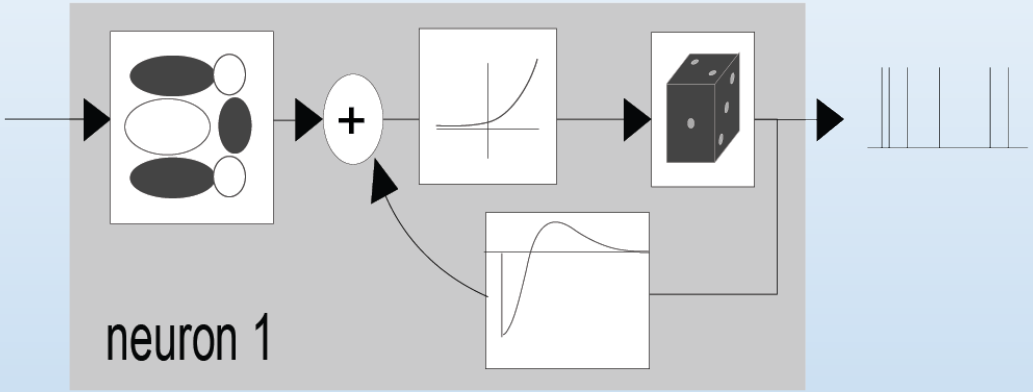
Meet the data



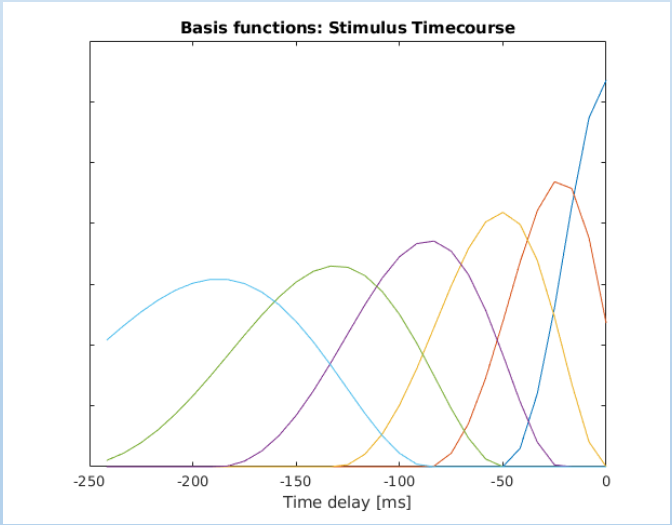
Meet the model

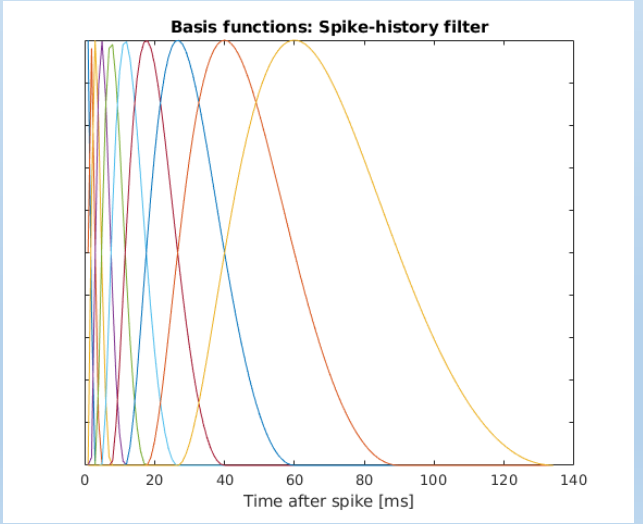
X

Rank 1 Matrix:
Stimulus x Time



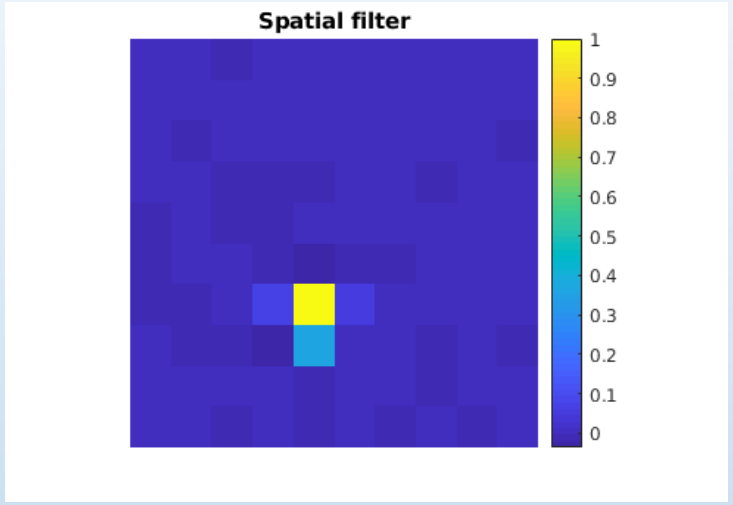
post-spike filter



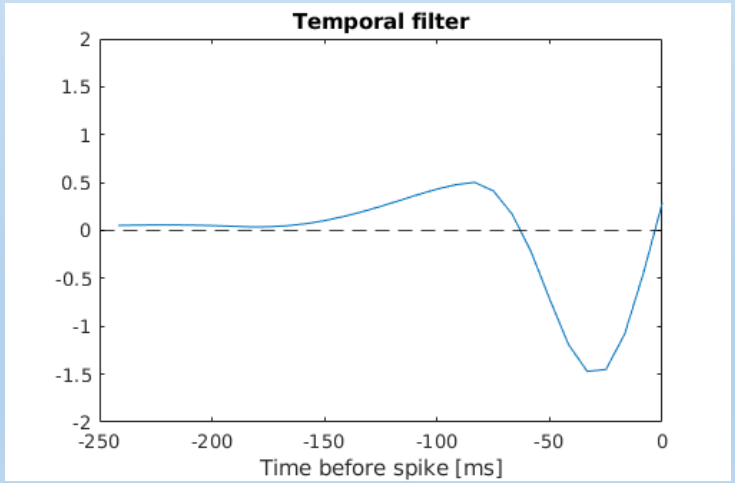


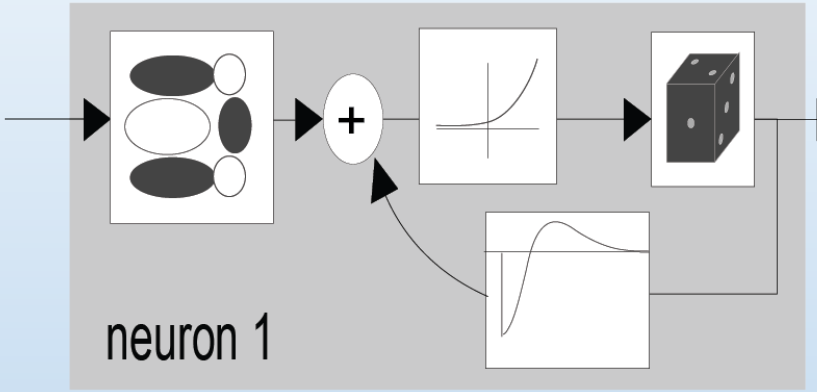
Code available at https://github.com/pillowlab/

Example fit

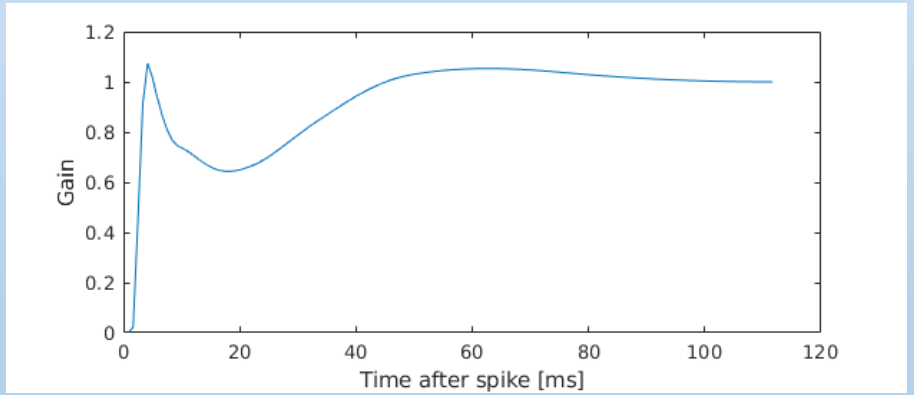


X

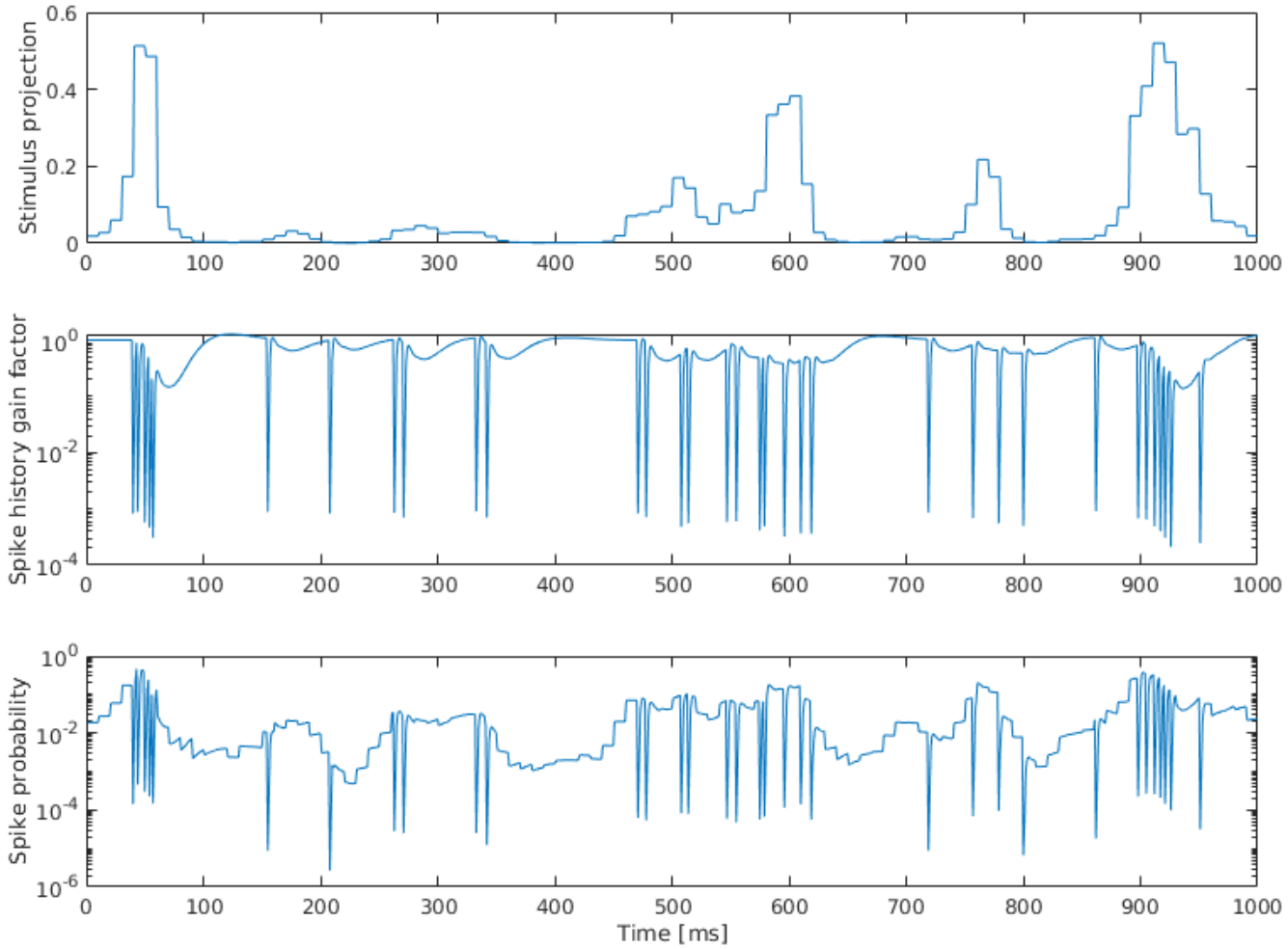




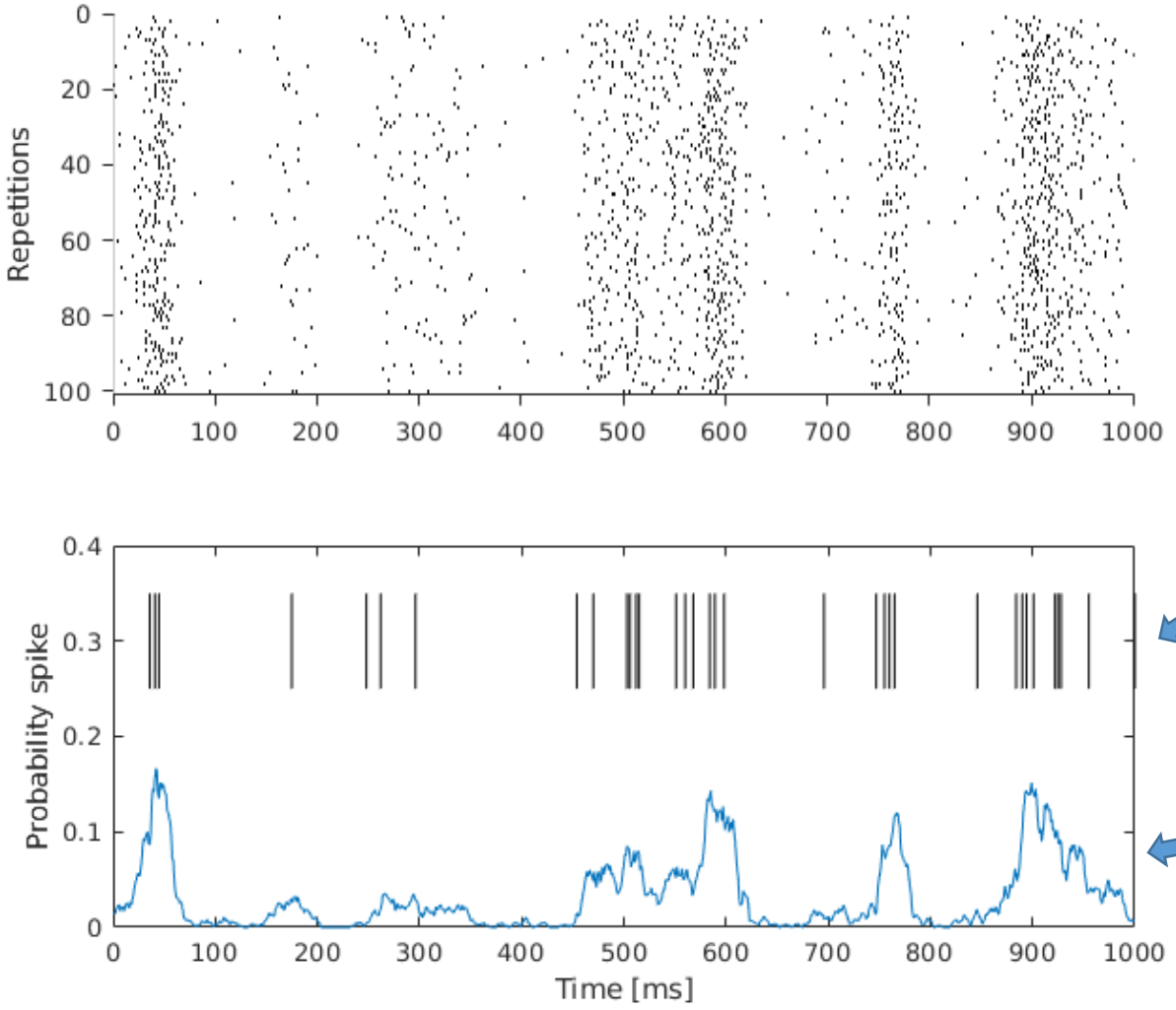
post-spike filter



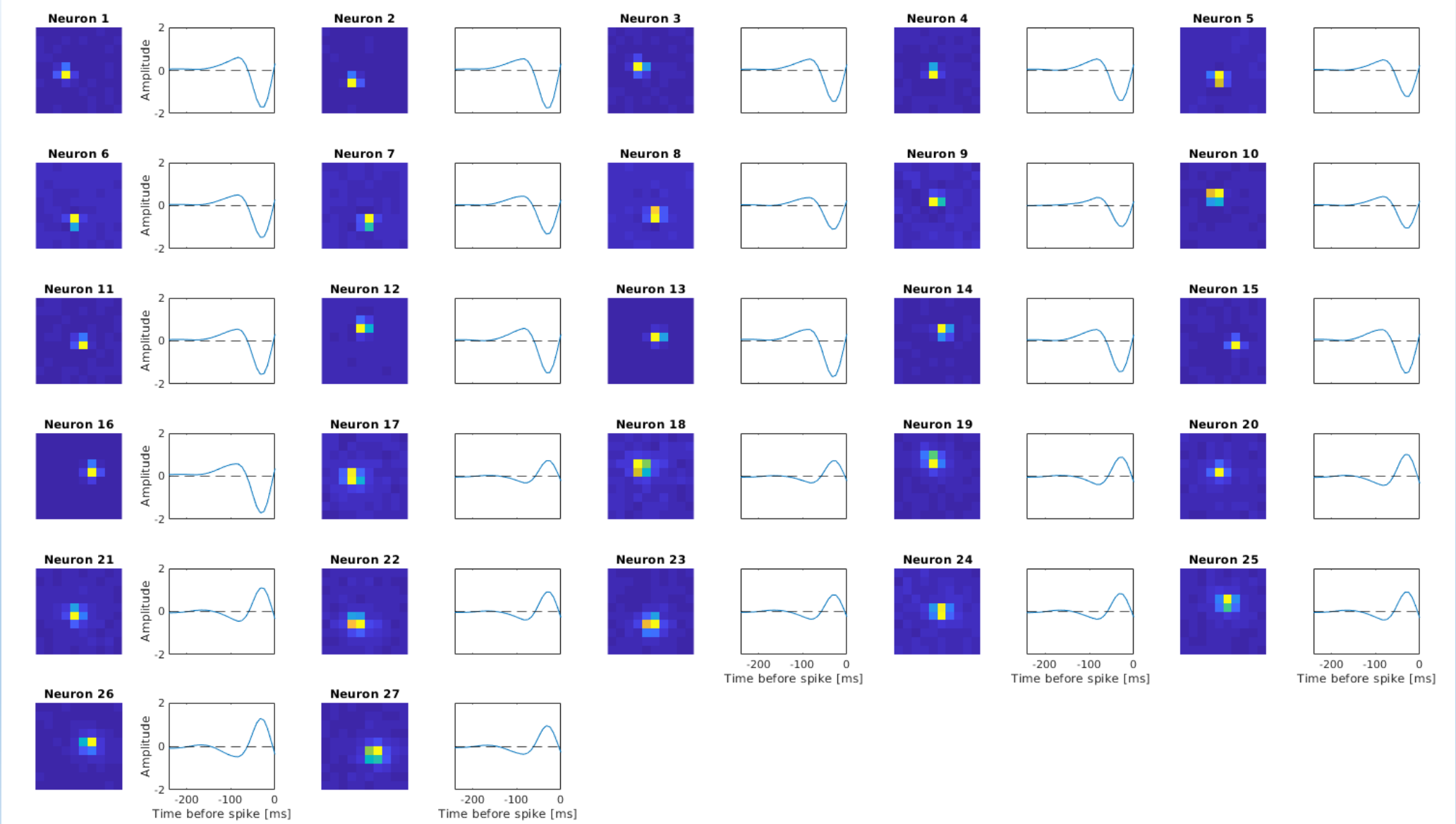
Example fit


Stimulus projection
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
800
900
1000
Spike history gain factor
10^{0}
10^{-2}
10^{-4}
Spike probability
10^{0}
10^{-2}
10^{-4}
10^{-6}
Time [ms]

Example fit


Repetitions
0
20
40
60
80
100
0
100
200
300
400
500
600
700
800
900
1000
Probability spike
0.4
0.3
0.2
0.1
0
Time [ms]
True spikes
Average predicted spike rate

Fit to all RGCs



Thank you!